ϕ = angle measured from stagnation point
ω_n = defined by Equation (2a)

Subscripts

c = critical condition
g = gas
l = liquid
m = maximum value
s = stagnation point

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Solids Mixing in Straight and Tapered Fluidized Beds

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It is generally thought that the solids in a gas fluidized bed mix rapidly and that this mixing is responsible for the desirable properties of fluidized beds, namely good contact between gas and solids, high rates of heat transfer, and temperature uniformity within the bed. But a high solids mixing rate is not always desirable. In the continuous chemical processing of solid materials (particularly where high conversion solid product is required) it is useful to reduce the rate of solids mixing in the longitudinal direction and still retain the desirable properties of the fluidized bed.

Solids mixing is not the only problem, however; deep beds of dense materials do not fluidize well. Levey et al. (3) reported violent bed eruptions and inefficient contacting in a 5-in. diameter bed of 20 to 40 mesh UO_3 particles when the static bed heights exceeded 4 diam. He observed that fluidization began at the upper surface of the bed and proceeded downward through the bed as the inlet gas velocity was raised, and when the gas velocity was sufficient to fluidize the bed completely, the upper portion of the bed was slugging.

Since the superficial gas velocity increases considerably along the bed owing to expansion of the gas (the pressure drop in beds of UO_3 is approximately 1.75 lb./sq. in./ft.) the bubble volume increases steadily along the bed, and thus the deeper the bed and the higher the particle density the greater the tendency of the bed to slug. To compensate for the gas expansion, Levey et al. tapered the bed to obtain a constant superficial gas velocity along the bed. In this way, solids mixing in the bed was reduced, deep beds of dense materials fluidized better, and both conversion rates and productivity were increased.

According to Romero and Johanson (9), the reduced rate of solids mixing is caused by operating close to the minimum fluidizing velocity (MFV). Sutherland (10) concluded that the effect of tapering is to reduce the vertical mixing rate in deep beds of dense materials. Furthermore, he specified that this effect is confined to beds in which the gas flow rates are less than 30% above the minimum and to bed heights greater than 2 ft. Sutherland found as Levey et al. did that particle movement and bubbling in untapered beds began at the top of the bed as the air rate was increased, and that by the time particle movement was observed at the base, the top was slugging violently. In tapered beds, he observed that the fluidization was more even with bubbles appearing in the lower as well as the upper part of the bed.

In this paper, solids mixing rates are reported for −140 +200 mesh copper particles in straight and tapered fluidized beds of rectangular cross section with a 2-in. sq. inlet. Gas velocities up to 110% above the minimum and bed height to diameter ratios of 8 and 16 to 1 were employed. The object of this work is primarily to compare the rates of solids mixing of dense particles in tapered and untapered beds with high L/D ratios (8 and 16 to 1).

EQUIPMENT AND MATERIALS

The apparatus consists of a metered air supply, two columns, counting, and recording equipment. A schematic drawing is shown in Figure 1.

The tapered and untapered columns used in this work are 50 in. high and have a 1.992 in. sq. inlet. The outlet for the tapered column is 1.992 × 2.880 in. The untapered column is assembled from four aluminum plates (¼ in. thick) which are gasketed and bolted together. When the tapered column is desired, the front and rear plates of the untapered column are replaced by two tapered plates, each making an angle of 0.509 deg. with the vertical.

The square inlet is bolted into a bottom header into which the metered gas is fed. Separating the gas space of the header from that in the column is a sintered, stainless steel, gas distributor plate. The outlet is fitted with a gas outlet port and a tracer charging mechanism. This mechanism consists of a cylinder, one end of which is soldered to a ⅜-in. hard copper tube and the other is closed by a rubber stopper. The copper tube passes through the top header where it is positioned. The tracer charging cylinder can thus be placed at any level in

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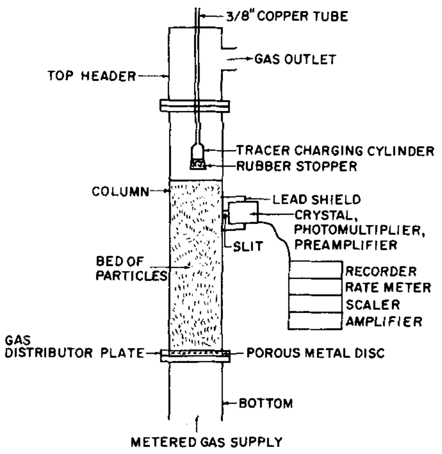

Fig. 1. Apparatus (schematic).

the column. The tracer is added through this tube to the charging cylinder.

Air is supplied to the lower header of the column through a pressure regulator and one of two rotameters of 1.2 and 6.0 std. cu. ft./min. capacity. A manometer measures the pressure drop across the bed and gas distributor plate and a scintillation counter with a sodium iodide crystal is used to detect the activity in the column. The crystal, photomultiplier, and preamplifier are fitted into a lead shield which has a ¼-in. horizontal slit 2 in. long arranged as shown in Figure 1. The thickness of the shield between the face of the crystal and the column well is 1⅛ in. The signal from the preamplifier feeds into a linear amplifier and then to a scaler and a rate meter. The output of the rate meter is read on a high-speed recorder.

Copper spheres in the —140 and 200 mesh size range were used in this work. The bulk density of these particles is 5.0 g./cc.

The tracer, copper 64, is obtained by irradiating for 30 sec. approximately 10 g. of copper spheres identical with those in the bed in the isotope tray (10^{12} flux) of the CP-5 reactor. This condition is the lowest flux-time combination available at CP-5, and the beads require a three day cooling period before use. The copper in each capsule had an activity of 10 to 20 mr./hr. (at 1 in. hard) on use. The copper has a 12.8 hr. half life which is close to ideal for these experiments.

The columns used in this work were square or rectangular in cross section rather than circular because the tapered column was easier to construct in rectangular geometry. There is some question as to how solids mixing rates in columns of rectangular cross section should be compared with those of circular cross sections. It is not possible to answer this question with any certainty considering the present state of knowledge of solids mixing and wall effects. In any case, however, the order of magnitude of the rate of mixing can be ascertained; the shape of the cross section of the column this writer believes cannot conceivably change the order of magnitude of the rate.

Column Taper

The usual method of calculating the required taper is to set as a condition that the superficial gas velocity through the bed remain constant as the pressure drops.

For a rectangular column with two tapered sides, when one considers isothermal flow through the bed, continuity and the gas laws require that

$$d(ap) = 0 \qquad (1)$$

The pressure drop at the MFV is

$$-dp = (1-\epsilon)(\rho_s - \rho_f)\frac{g}{g_c}dz \qquad (2)$$

When one combines Equations (1) and (2) and assumes the gas is ideal, the pressure variation along the column is

$$p = \left(\frac{RT}{M}\rho_s\right) - \left(\frac{RT}{M}\rho_s - p_o\right)e^{-(1-\epsilon)\frac{M}{RT}\frac{g}{g_c}(L-z)}$$

The taper angle θ is

$$\theta = \tan^{-1}\left[1/2\frac{da}{dz}\right] \qquad (4)$$

Equations (1) and (3) are used to evaluate Equation (4). The result is then simplified into its final form by replacing the exponential terms by the first two terms of their power series since the exponent is small. The taper angle for the ideally tapered bed is

$$\theta = \tan^{-1}\left[\frac{a_o \rho_B}{2\,p_o}\right] \qquad (5)$$

The equation has little meaning as soon as bubbles appear in the bed because the bed is no longer homogeneous as assumed in the derivation. In addition Equation (2) is not strictly valid either except close to the MFV.

These studies were made with an undertapered column with a taper of 0.509 deg., whereas the required taper from Equation (5) is 0.685 deg. Therefore, the gas velocity always increased slightly from bottom to top, but the increase was far less than that in the untapered bed.

PROCEDURE

The bed is fluidized and the flow rate adjusted to that desired for the run. While the system is reaching a steady state, approximately 10 g. of copper beads are added to the tracer charging cylinder. The total activity of the charge is 10 to 20 mr./hr. (at 1 in. hard). The activity of the beads in the charging cylinder is measured by the counter and the rate meter scale adjusted accordingly. The counter is then set at the desired location for the run. The amplifier, scaler, rate meter, and recorder are allowed to warm up and the initial bed activity recorded.

The run is started by knocking the rubber stopper out of the charging cylinder with a long rod. This allows the tracer to enter the fluidized bed. Simultaneously with the tracer addition, the recorder is started and allowed to run until a steady state is re-established in the column.

Code for Runs

Each series of runs is coded as follows. The letter *T* or *U* designates whether the bed is tapered or not. The numbers which follow designate the nominal bed height 16 or 32 in., the superficial gas velocity at the inlet and the location of the counter. *U*-16-0.197-0.324*L* designates that the run was made in the untapered bed with a nominal bed height of 16 in.; the inlet gas velocity was 0.197 ft./sec., and the counter was located 0.324*L* in. below the top of the bed. *L* is the length of the bed.

RESULTS

Pressure Drop and the Minimum Fluidizing Velocity

Untapered Bed. The MFV in untapered beds of copper particles, 16 and 32 in. deep as used in this study, is ambiguous because these beds fluidize progessively from top to bottom as the flow rate is increased. In such beds, as Sutherland (*11*) has noted, there is no exact value for MFV. Therefore, in the present study, three criteria as described below were used to ascertain approximately when the MFV was reached.

The gas velocity obtained graphically from the intersection of the fixed bed portion of the pressure drop-flow rate curve (obtained by lowering the flow rate through an already fluidized bed) and the calculated horizontal line representing the bed weight per unit area is 0.094 ft./sec. (29.1 lb./hr. sq. ft.) in the 16 in. deep bed; in the 32 in. deep bed the value is 0.092 ft./sec. (33.4 lb./hr. sq. ft.). At these gas velocities, the beds are only partially fluidized as determined by probing with a thin rod. It is

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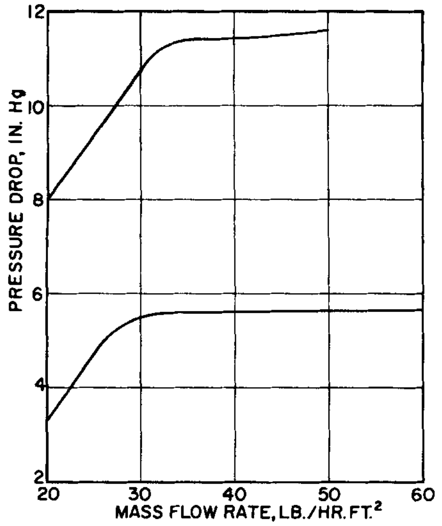

Fig. 2. Pressure drop—flow rate curves in untapered bed.

also below the velocity at which solids mixing takes place at a rate such that tracer particles charged into the bed will become uniformly distributed through the bed in less than 60 min. In the 16-in. bed this condition begins at a gas velocity of about 0.099 ft./sec. and in the 32 in. deep bed the velocity is higher but less than 0.111 ft./sec.

The pressure drop flow rate curves for the 16 and 32 in. deep beds are given in Figure 2. The graph for the 32 in. deep bed is similar to that described by Sutherland (*10*). It deviates from linearity well before the bed is fluidized, and the horizontal linear portion does not exist. At the 0.185 ft./sec. inlet gas velocity (52.8 lb./hr. sq. ft.) the bed was slugging and manometer oscillations were so large that the pressure drop could not be read. The 16 in. deep bed is more normal. At the 0.197 ft./sec. inlet gas velocity (48.5 lb./hr. sq. ft.), the manometer oscillations are ± 0.2 to 0.3 in. Hg. The pressure drop was taken as the average of the lowest and highest fluctuation read.

Tapered Bed. At the MFV, the bed is acting as a fluid and the pressure drop is calculated by the well-known equation

$$\Delta P = (1-\epsilon)(\rho_s - \rho_f)\frac{g}{g_c} L = \frac{W}{A_{av}} \qquad (6)$$

The gas velocity obtained graphically from the intersection of the fixed bed portion of the pressure drop-flow rate curve (obtained by lowering the flow rate through a bed already fluidized) and a horizontal line representing the pressure drop as calculated from Equation (6) is 0.094 ft./sec. (29.3 lb./hr. sq. ft.) in the 16 in. deep bed and 0.099 ft./sec. (35.8 lb./hr. sq. ft.) in the 32 in. deep bed. Levey (*4*) has found that in ideally tapered beds this graphical procedure gives the MFV because the results were in accord with rod probing experiments.

The above graphical procedure for determining the minimum fluidizing velocity in an ideally tapered bed is analogous to the method used by van Heerden, Nobel, and van Krevelen (*13*) for untapered beds and should give the MFV so long as the bed does not fluidize progressively from top to bottom as the gas velocity is raised. It is of interest to note, however, that in the 32 in. deep bed the gas velocity at which uniform dispersion of the tracer particles occurs in less than 60 min. is above 0.113 ft./sec.

The pressure drop-flow rate curves in the tapered bed are like those ordinarily encountered in fluidized beds. There is a linear region before the bed is fluidized, and there is a horizontal linear portion. However, at the highest inlet gas velocities used manometer fluctuations of ± 0.2 in. Hg were obtained in the 32 in. deep bed.

The Measured Activity

It is desirable to know what the measured activity represents in terms of contributions made by portions of the bed at various distances from the detector slit. This can be done only approximately because of the complexities of gamma attenuation calculations. Such an approximation is, however, adequate to show that mixing could not occur undetected in any part of the bed and that the measured activity includes substantial contributions from the central portions of the bed.

Determination of Steady State and Bed Height

The solids mixing data consist of activity-time traces. Each trace represents a run in which the gas velocity and counter location are fixed, and the activity is recorded as a function of time on a strip chart until a steady state is reached. The trace is not a smooth line. The instantaneous activity fluctuates widely so that the traces had to be smoothed. This was done by eye with the counts taken on a scalar in several time intervals during the run as a guide.

Because of the fluctuations in the activity recorded on the strip chart, it requires about 180 to 300 sec. to judge that the activity has become steady at a given location. This judgment is made first by observation of the chart and confirmed by a series of counting measurements over 1-min. time intervals with the scaler. Even after this amount of time it is difficult to distinguish a true steady state from an apparent one drifting slowly toward the true one because of statistical problems involved in counting measurements. Therefore, after it appears that a steady state has been reached at a given location, a longitudinal activity profile is obtained to see if the activity is uniformly distributed throughout the bed. This is done by counting

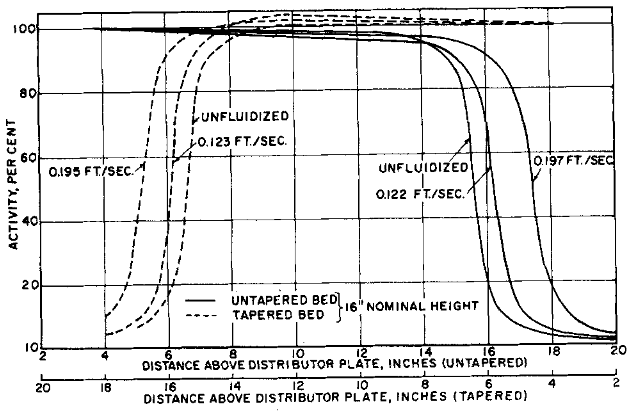

Fig. 3. Longitudinal activity profile, 16-in. nominal bed height.

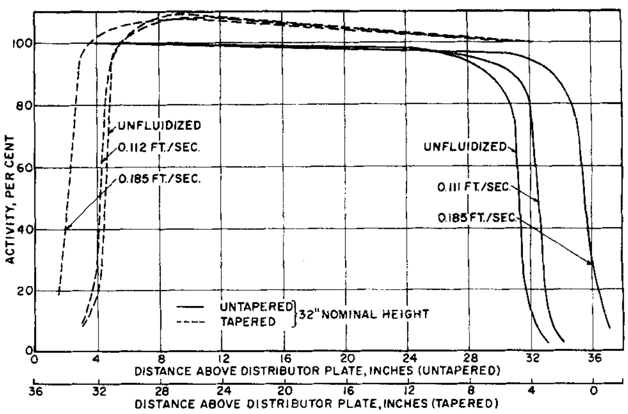

Fig. 4. Longitudinal activity profile, 32-in. nominal bed height.

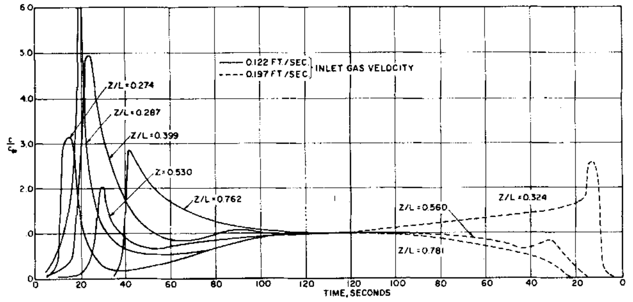

Fig. 5. Activity—time curves, untapered bed, 16-in. nominal bed height.

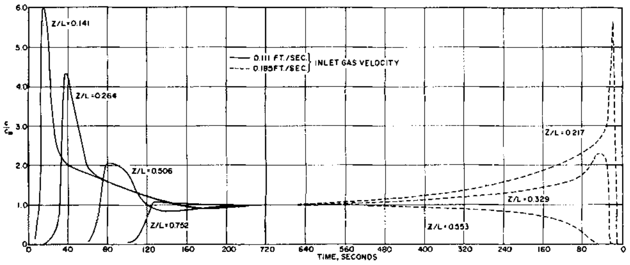

Fig. 6. Movement of activity front in tapered and untapered beds.

for 1 or 2 min. at each location. Typical longitudinal activity profiles at steady state are given in Figures 3 and 4. When a measured profile is obtained, which is the same as the typical ones, the bed is considered to have reached a true steady state. In some runs, however, the activity on the strip chart appeared to be steady for a few minutes but the activity was not uniformly distributed throughout the bed. This occurred, for example, in the 16 in. deep bed for the 0.0985 inlet gas velocity. Longitudinal activity profiles were also used to determine the height of the fluidized bed which was taken to be that height at which the activity was one-half the difference between the activities of the flat portions of the profiles taken in the region of steepest descent. This method was found satisfactory for the unfluidized bed, and its use was assumed to apply to the fluidized bed.

The column could not be scanned at locations less than 3.9 in. above the distributor plate because the shield around the scintillation counter bumped into the flange into which the column was bolted. In cases where it was necessary to know if the activity had reached and become uniform in that region, portable radiation monitors were used. These monitors are, of course, much less sensitive than the scintillation equipment and only gross differences in activity could be recognized when using them.

Reproducibility of an Individual Run

Duplicate runs are reproducible in the sense that time features of the run such as the time at which the activity first reaches a given location (defined as the time at which the normalized activity c/c_∞ is 0.1 at that location) or the time at which an activity peak or steady state occurs is reproducible to within 10%. The magnitude of the normalized activity at a given time is not reproducible to the same degree. The tail end of the curve for runs exhibiting an activity peak and the entire curve for runs without activity peaks are reproducible to within 20%, but activities are not reproducible at peaks and in regions where the activity-time curve is steep. This is probably because solid particle flow patterns are not reproducible as Marcheck and Gomezplata have found (*8*).

Effect of Gas Velocity in the U-16 Bed

The inlet gas velocity of 0.0985 ft./sec. is close to the velocity at which the entire bed is fluidized and mixing is very slow. It takes about 1,460 sec. to distribute the activity uniformly in the bed.

At this gas velocity, the rate of mixing decreases with distance from the top of the bed. This was determined from the runs in which the z/L ratios were 0.261 and 0.519. When the activity appeared to become steady (at $z/L = 0.261$ after about 600 sec. and at $z/L = 0.519$ after about 1,100 sec.), the longitudinal activity profile was measured. It revealed that above the initial counter location the activity was substantially constant, while below that location the activity dropped off with distance from the counter. Clearly, the mixing had to be faster above the initial counter location than below it.

When the inlet gas velocity is raised to 0.122 ft./sec., the rate of mixing is increased considerably. The bed came to steady state in about 120 sec. The time to reach steady state was not changed by increasing the gas velocity to 0.197 ft./sec.

The normalized activity-time curves for the *U*-16-0.122 and *U*-16-0.197 runs are very different in shape (Figure 5). Peaks are lower at the higher gas velocity, and no minima or cycling are observed in the traces. The normalized activity of the maxima in the *U*-16-0.122 runs vary irregularly, but the times of their appearance are in the expected order.

The time required for the activity front to be detected at a given location in the bed is given in Figure 6. The front velocity in the top 7 in. of the 16-in. bed is unchanged as the gas velocity is increased from 0.122 to 0.197 ft./sec. It averages 1.0 in./sec. (0.083 ft./sec.) in traversing the top 5 in. of the bed. In the bottom half of the bed, the front velocities are reduced to 0.18 and 0.40 in./sec., respectively. Extrapolation of the *U*-16 curves in Figure 6 reveals that the activity reaches the bottom of the bed at the 0.122 and 0.197 ft./sec. gas velocities in 54 and 36 sec., respectively.

Effect of Gas Velocity in the U-32 Bed

The most striking effect of the gas velocity in the 32-in. bed is that the time required for steady state is much smaller at the lower gas velocity. A steady state is obtained in about 230 sec. at the 0.111 ft./sec. gas velocity, while it takes 660 sec. at 0.185 ft./sec.

The bed is slugging at the higher gas velocity, and it appears that this slugging reduces the number of particles passing between levels at the top of the bed and thereby increases the time required to obtain a uniform activity in the bed.

In Figure 6, however, it is seen that the activity front passes through the bed faster at the higher gas velocity. In the top 5 in. of the bed, the front velocity is about 1 in./sec. Five to twelve inches below the top of the bed, the front velocity is higher at the higher gas velocity, and below the first 12 in. of the bed the front velocities are about 0.2 in./sec. at both gas velocities. If the *U*-32 curves of Figure 6 are extrapolated, the activity reaches the bottom of the bed in about 140 sec. at both gas velocities. The major difference between these results and those in the shorter bed is at the higher gas velocity where the activity

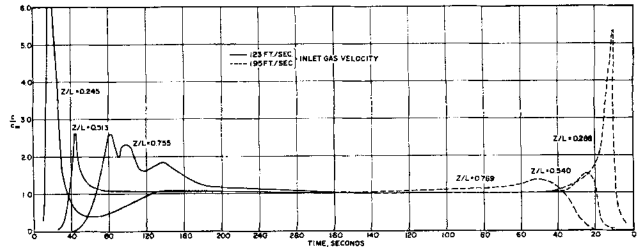

Fig. 7. Activity—time curves, untapered bed, 32-in. nominal bed height.

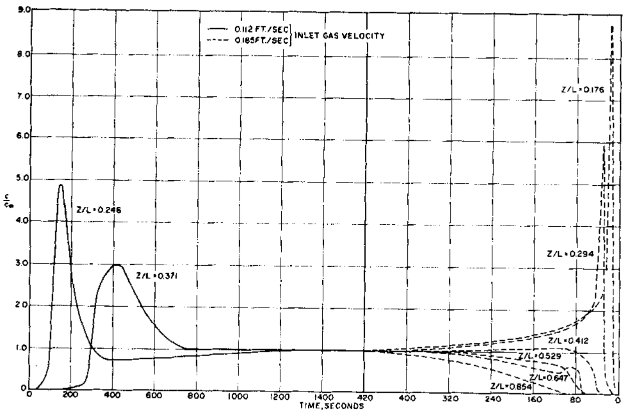

Fig. 8. Activity—time curves, tapered bed, 16-in. nominal bed height.

front velocity is larger in the bottom of the *U*-16 than the *U*-32 bed.

The shapes of the activity-time curves (Figure 7) are affected by the gas velocity as found in the *U*-16 bed. Activity peaks are higher at the lower gas velocity, but the minima are not as pronounced as in the *U*-16 bed.

Effect of Gas Velocity in the T-16 Bed

Increasing the inlet gas velocity in the *T*-16 bed from 0.123 to 0.195 ft./sec. reduces the time required to attain uniform activity in the bed from 316 to 160 sec. In the tapered bed the time required for the first activity to reach the bottom of the bed is reduced from 69 to 31 sec. by raising the gas velocity. Thus, the taper does reduce the rate of solids mixing significantly at the lower gas velocity.

The activity-time curves at both gas velocities (Figure 8) give the appearance of reaching steady state in the top of the bed faster than at the bottom. What is happening in these runs is that the mixing rate is not uniform throughout the bed but much faster at the top than at the bottom. The slow drift toward steady state in runs *T*-16-0.123-0.245*L* and 0.513*L* in Figure 8 after 135 sec. results from slow mixing in the bottom of the column. A similar observation can be made at the higher gas velocity.

Increasing the inlet gas velocity from 0.123 to 0.195 ft./sec. increases the rate of mixing in the bed; yet it is also observed that the rate of mixing is higher at the top of the bed than at the bottom where the superficial gas velocities are practically the same. The reason for this is that bubbles cause solids mixing and that bubble size and distribution are not determined at any point in the bed solely by the superficial gas velocity.

The average velocity of the front in the top 5 in. of the bed increases from 0.36 to 1.67 in./sec. as the inlet gas velocity is raised from 0.123 to 0.195 ft./sec. Below the top 5 in. of the bed the front velocity at the higher gas velocity is 0.44 in./sec., while at the lower gas velocity it diminishes slowly as the activity penerates the bed and becomes 0.19 in./sec. at 12 in. below the top of the bed. Raising the gas velocity increases the front velocity at all levels in the bed. In the untapered bed, only the front velocity in the bottom half of the bed was affected by the change in gas velocity. The front velocity at the higher gas velocity is the same in the tapered and untapered beds, while at the lower gas velocity the tapered bed has the lower front velocity.

The major effect of the taper in the 16 in. deep bed is to reduce solids mixing in the lower part of the bed.

Effect of Gas Velocity in the T-32 Bed

The activity does not become uniform in the bed in 60 min. at the 0.112 inlet gas velocity. This gas velocity is above the minimum of 0.099 ft./sec. which was obtained by the method of van Heerden, Nobel, and van Krevelen (*13*) applied to a tapered bed. There is solids mixing in the bed at all levels, but the rate of mixing in the bottom six inches of the column is very slow. Above this region the mixing is more rapid but still slower than at the 0.185 ft./sec. gas velocity. In the runs *T*-32 to 0.112 to 0.245*L* and *T*-32 to 0.112 to 0.381*L* the activity changes slowly after about 1,000 sec. to reflect the slow mixing near the bottom of the column (Figure 9). At all locations more than 20 in. above the distributor plate, the activity was found uniform when measured 1,000 sec. after the run was begun.

The front velocity in the top 5 in. of the bed is 1.67 in./sec. about the same as in the *T*-16 bed at the 0.195 ft./sec. gas velocity. The activity front slows down to 0.19 in./sec. 10 in. below the top of the bed. The front movement in the *U*-32 and *T*-32 beds at the higher gas velocity are about the same.

At the 0.185 ft./sec. gas velocity, the bottom of the bed does not appear to mix differently than the top as was observed in the *T*-16 bed. The activity in the bed does become uniform faster in the *T*-32 to 0.185 runs than in the *U*-32 to 0.185 runs, indicating that tapering increases solids mixing in a bed that is slugging but has no effect on the velocity of the activity front.

DISCUSSION

The object of this work is primarily to compare the rates of solids mixing in tapered and untapered beds with high *L/D* ratios (8 and 16 to 1). Two indexes of the rate of solids mixing, namely the time required to uniformly disperse the activity charged to the bed and the velocity of the activity front, have been used in this study. There are no adequate theories to describe either the rate of solids mixing or particle flow patterns in a fluidized bed. This makes a discussion of the data difficult. In this section, the important effects of tapering and the comparison of the observed rates with those found in other experiments are given.

In the 16 in. deep bed, the taper reduces the rate of solids mixing significantly at the 0.123 ft./sec. inlet gas velocity (30% above the minimum). Both the activity front velocity and the time required to disperse the activity uniformly are reduced principally because of the slow mixing in the bottom of the tapered bed. These observations are not totally in agreement with Sutherland (*10*) because the beds were less than 2 ft. in height. They do confirm, however, the suggestion by Romero and Johanson (9) that the reduced rate of solids mixing found by Levey and co-workers (3) in a tapered bed is the result of working close to the minimum fluidizing velocity.

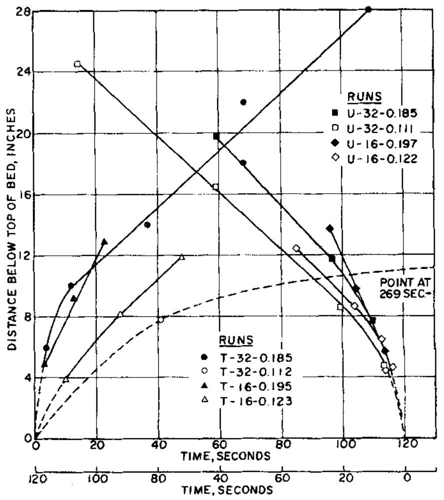

Fig. 9. Activity—time curves tapered bed, 32-in. nominal bed height.

A second effect of importance found in the 32 in. deep bed is that the slugging observed in the untapered bed at the 0.185 ft./sec. gas velocity slows the rate at which solids entering the bed are uniformly dispersed in it. Tapering had the effect of reducing slugging and increasing the mixing at the top of the bed. At the 0.111 ft./sec. gas velocity tapering reduced solids mixing because of slow mixing in the bottom of the bed. The velocity of the activity front was unaffected by tapering the bed at the higher gas velocity but was substantially reduced at the lower gas velocity.

The range of front velocities found in this work are in the range reported by Leva and Grummer (*1*) for sand and silica gel. They are lower than those obtained by photographic techniques (*6, 12*) by 1 order of magnitude.

Two solids mixing studies are similar to those reported in this work. Levey et al. (*3*) performed batch solids mixing experiments using UO_3, UO_2, and UF_4 in cylindrical and tapered units. Their model for mixing is the one dimensional diffusion equation with a constant diffusivity and no flux end conditions. Diffusivities in 4- and 5-in. diameter beds up to 72 in. high ranged from 4 to 20 sq. ft./hr. in untapered cylindrical beds and 0.1 to 0.4 sq.ft./hr. for the tapered units for −20 +40 mesh particles at gas velocities 10 to 20% above the minimum.

May's (*7*) solids mixing experiments were performed in large diameter beds using cracking catalyst in the size range 20 to 150μ. His model for mixing is essentially the same as Levey used, and mixing was followed by radioactive tracer techniques using Iodine-132 as the tracer. Solids diffusivities of about 0.6, 1.5, and 5 sq.ft./sec. were obtained in 3-, 15-, and 60-in. diameter beds 32 ft. deep with a superficial gas velocity of 0.8 ft./sec. which is far above the minimum. Interpolating May's data to obtain an estimate of the diffusivity in a 5-in. diameter column (2,700 sq.ft./hr.) one finds his diffusivity is a few hundred times greater than that of Levey et al.

Massimilla and Bracale (*5*) report solids mixing diffusivities using 0.7-mm. glass particles in a 90-mm. column. Solids mixing coefficient increased with gas velocity from 1.07 sq.ft./hr. at a mass velocity 380 lb./hr. sq.ft. to 1.82 sq.ft./hr. at a velocity 445 lb./hr. sq.ft. The mass velocity at minimum is estimated at 310 lb./hr. sq.ft. from Leva's correlation (*2*), so that the mixing studies were conducted at gas velocities not far above the minimum.

In order to compare the results of this work with those described above, solids mixing diffusivities were calculated from the data where possible. The diffusivities obtained in the *U*-16-0.197 and *U*-32-0.185 data are 0.005 and 0.004 sq.ft./sec. (18.0 and 14.4 sq.ft./hr.). This is in the range which Levey reported. The cracking catalyst data were taken at gas velocities fifty times above the emulsion gas velocity, and it is possible that some of the differences in diffusivities are attributable to this factor. Another factor is that in the large diameter columns used by May, the measured activity represented mainly the activity near the wall.

The tapered bed diffusivity in the *T*-32-0.185 runs, 0.005 sq.ft./sec. (18.0 sq.ft./hr.), is substantially above that found by Levey et al. However, Levey's results were obtained at gas velocities 10 to 20% above the minimum, whereas the 0.185 ft./sec. velocity is considerably above that figure and the differences are attributed to this fact.

The diffusion theory did not apply to some of the data, and the reported diffusivities simply give a qualitative measure of the solids mixing rate. Solids flow patterns in fluidized beds are undoubtedly complicated. The only measurements reported to date are those of Marcheck and Gomezplata (*8*) who found radial variations in the particle flow pattern dependent on the superficial gas velocity and the z/L ratio.

No useful comparison of these data can be made with those of Talmor and Benenati (*11*) who reported solid circulation rates based on a model described by Zenz and Othmer (*14*).

CONCLUSIONS

For the deep beds of dense particles used in this study, tapering a fluidized bed slows axial solids mixing at gas velocities close to the minimum primarily by reducing solids mixing in the lower part of the bed. Substantially slower mixing was observed at gas velocities 30% above the minimum, and this effect was more pronounced at gas velocities closer to the minimum.

Slugging reduces the rate at which particles entering the bed are uniformly dispersed in it. Tapering a bed that slugs increases the dispersion rate principally by increasing the mixing rate at the top of the bed.

NOTATION

a = distance perpendicular to the longitudinal axis of the column between tapered sides
A_{avg} = average cross-sectional area of the bed
c = concentration or activity
c_∞ = concentration or activity at steady state
g = acceleration due to gravity
L = length of the bed
M = molecular weight of gas
p = pressure
R = gas constant
T = absolute temperature
W = bed weight
z = column length variable
ϵ = void fraction
θ = taper angle
ρ = density

Subscripts

avg = average
B = bulk
f = fluid
O = at inlet
s = solids

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